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Determination of the width of the output angular power distribution in step-index multimode optical fibers

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Abstract
Two functions for calculating the width of the output angular power distribution are tested for light launched centrally along the axis of a step-index plastic optical fiber. It is found that the more recent of the two agrees better with experimental measurements. The other function (Gloge's) underestimates the output width for longer fiber lengths, which is attributed to it not accounting for an appropriate boundary condition at the critical propagation angle.

Keywords: step-index optical fibers, mode coupling

1. Introduction

Transmission characteristics of step-index (SI) optical fibers depend strongly upon the differential mode attenuation and the rate of mode coupling. The latter represents power transfer from lower to higher-order modes caused by fiber impurities and inhomogeneities introduced during the fiber manufacturing process (such as microscopic bends, irregularity of the core–cladding boundary and refractive index distribution fluctuations). In the absence of these intrinsic perturbation effects (an idealized situation), light launched at a specific (non-zero) angle θ with respect to the fiber axis could be imaged into a thin ring by the far-field output of the fiber end. The ring diameter corresponds to the said launch angle θ. Due to inevitable mode coupling in real situations, the boundaries of such a ring blur for longer fibers. This fussiness increases with fiber length until it eventually evolves into a disc for fibers longer than the 'coupling length' L_c. The 'equilibrium mode distribution' (EMD) exists beyond the coupling length L_c of the fiber. It is characterized by the absence of rings regardless of the launch conditions, even though the disc pattern may have different light distributions across it depending on the launch conditions. EMD indicates a substantially complete mode coupling and is of critical importance when measuring characteristics of multimode optical fibers (linear attenuation, bandwidth, etc). Indeed, measurement of these characteristics would only be considered as meaningful if performed at the EMD condition when it is possible to assign to a fiber a unique value of loss per unit length [1].

By further increasing the fiber length to z_s ($z_s > L_c$), all individual disc patterns corresponding to different launch angles take the same light distribution and the 'steady state distribution' (SSD) is achieved. SSD indicates the completion of the mode coupling process and the independence of the output light distribution from launch conditions.

For light launched centrally along the axis ($\theta = 0$), a point/disc radiation pattern is formed at the output fiber end. This disc broadens with fiber length as more of the higher-order modes are excited by the coupling which is more pronounced in longer fibers. Because the excitation of higher-order modes

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influences transmission characteristics of optical fibers, it is of interest for one to be able to predict the width of the output angular power distribution at the end of the fiber up to the fiber length z_s where steady state distribution is achieved.

Said in other words, fiber imperfections cause the energy packets to randomly switch modes back and forth and the modes are no longer independent. While this degrades the beam quality, its positive aspect is that it tends to average the propagation velocity for different modes, thus reducing the broadening of transmitted signal pulses. This broadening is a linear function of fiber length z before the EMD condition ($z < L_c$), but only a square-root function thereafter. Consequently, fiber bandwidth for longer fibers ($z > L_c$) drops as $1/z^{1/2}$, which is an improvement over the more rapid drop of $1/z$ for $z \leqslant L_c$ [2, 7].

Output angular power distribution in the near-and far-fields of an optical fiber end has been studied extensively. Work has been reported using geometric optics (ray approximation) to investigate mode coupling and predict output-field patterns [2, 3]. By employing the power flow equation [4–9] as well as the Fokker–Planck and Langevin equations [10], these patterns have been predicted as a function of the launch conditions and fiber length. We recently proposed a method for calculating the width of the output angular power distribution in step-index multimode optical fibers for light launched centrally along the fiber axis [11]. This method is now verified specifically for plastic optical fibers (POFs) by reference to published experimental data by Mateo *et al* [12] and our experimental data from the same experiment. It is also compared to Gloge's analytical solution [5].

2. Calculation of the width of the output angular power distribution

Assuming that mode coupling in multimode optical fibers occurs predominantly between adjacent modes, Gloge has derived the time-independent power flow equation in the following form [5]:

$$\frac{\partial P(\theta,z)}{\partial z} = -A\theta^2 P(\theta,z) + \frac{D}{\theta}\frac{\partial P(\theta,z)}{\partial\theta} + D\frac{\partial^2 P(\theta,z)}{\partial\theta^2} \quad (1)$$

where $P(\theta,z)$ is the power distribution in the fiber, θ is the propagation angle with respect to the core axis, z is the coordinate along that axis referenced to the input fiber end, $D = d_0(\lambda/4an)^2$, λ is the free-space wavelength, c is velocity of light in vacuum, a and n are core radius and refractive index, respectively, d_0 is the zero-order term of the expression for the coupling coefficient $d(\theta) = d_0 + \alpha(\theta)\theta^2 + \cdots$ and A is the second-order multiplicative factor in the series expansion of the power loss coefficient $\alpha(\theta)$ due to absorption and scattering: $\alpha(\theta) = \alpha_0 + A\theta^2 + \cdots$. The tacit assumption here that the coupling coefficient $d(\theta)$ is constant (equal to d_0) has been made routinely in the absence of reliable estimates of other terms in the expansion series of $d(\theta)$ [4, 11, 13, 14].

With θ_c denoting the critical angle of the fiber, the boundary conditions for (1) are $P(\theta_c,z) = 0$ and $D\cdot\partial P/\partial\theta = 0$ at $\theta = 0$. The condition $P(\theta_c,z) = 0$ implies that modes with infinitely high loss do not carry power. Condition $D\cdot\partial P/\partial\theta = 0$ at $\theta = 0$ indicates that the coupling is limited to modes propagating with $\theta > 0$.

In the case of a Gaussian input beam distribution launched along the fiber axis:

$$P_{in} = P_0 \exp\left[-\frac{\theta^2}{\Theta_0^2}\right] \quad (2)$$

where $\Theta_0 = \sqrt{2}\sigma_0$ is the width of the Gaussian launch beam distribution (σ_0 is the standard deviation of the launch beam distribution).

Neglecting the boundary condition $P(\theta_c,z) = 0$, the analytical solution of equation (1) is [5]

$$P(\theta,z) = \frac{P_0\Theta_0^2}{\Theta_\infty^2 \sinh\gamma_\infty z + \Theta_0^2\cosh\gamma_\infty z}\exp\left[-\frac{\theta^2}{\Theta^2(z)}\right] \quad (3)$$

where $\Theta(z) = \sqrt{2}\sigma(z)$ is the width of the Gaussian angular power distribution at the end of the fiber length z, $\Theta_\infty = (4D/A)^{1/4}$ is angular width of the steady state angular power distribution and $\gamma_\infty = (4DA)^{1/2}$ is the overall loss coefficient. Angular width of the output angular power distribution at the end of the fiber length z can be obtained from [5]

$$\Theta^2(z) = \Theta_\infty^2\frac{\Theta_0^2 + \Theta_\infty^2\tanh\gamma_\infty z}{\Theta_\infty^2 + \Theta_0^2\tanh\gamma_\infty z}. \quad (4)$$

Gloge has shown that, if the measured width $\Theta(z)$ is small compared to the steady state width Θ_∞, equation (4) can be approximated as

$$\Theta^2(z) = \Theta_0^2 + \Theta_\infty^2\gamma_\infty z, \quad (5)$$

i.e.

$$\Theta^2(z) = \Theta_0^2 + 4Dz. \quad (6)$$

Since $\Theta_0 = \sqrt{2}\sigma_0$ and $\Theta(z) = \sqrt{2}\sigma(z)$, equation (6) can be written in the form:

$$\sigma^2(z) = \sigma_0^2 + 2Dz. \quad (7)$$

By independent calculations, in our previous work [11] we have shown that the variance of the Gaussian angular power distribution at the end of the fiber length z is given in the form of equation (7). Namely, except near cutoff, $A\theta^2$ need not be accounted for when solving (1) for mode coupling, and the first term on the right-hand side can be omitted. Additionally, when the launch distribution at the input end of the fiber is centered at $\theta_0 = 0$, due to the boundary condition $D\cdot\partial P/\partial\theta = 0$ at $\theta = 0$, equation (1) further reduces to [11]

$$\frac{\partial P(\theta,z)}{\partial z} = D\frac{\partial^2 P(\theta,z)}{\partial\theta^2}. \quad (8)$$

With distance from the input fiber end, the peak of the power distribution remains at zero angle but the width of the distribution increases. If one thinks of $P(\theta,z)$ as a probability distribution, equation (8) can then be seen as the special Fokker–Planck equation with constant diffusion coefficient D [11, 15]. The solution of equation (8) is [15]

$$P(\theta,z) = \frac{1}{\sqrt{2\pi}\sigma(z)}\exp\left[-\frac{\theta^2}{2\sigma^2(z)}\right]. \quad (9)$$

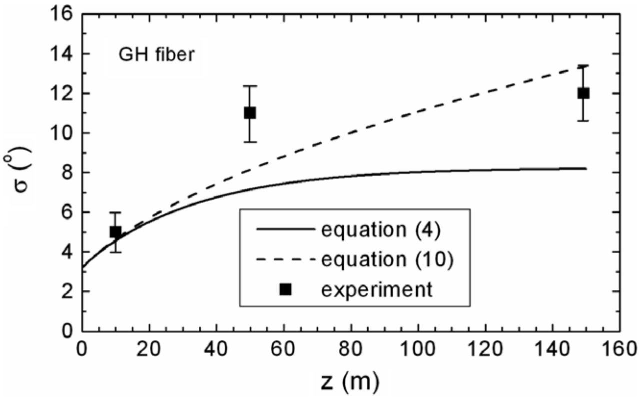

Figure 1. Standard deviation of the output angular power distribution as a function of fiber length in GH fiber calculated by Gloge's function (4) (solid line) and by our recently proposed function (10) (dashed line). Filled squares represent experimental data.

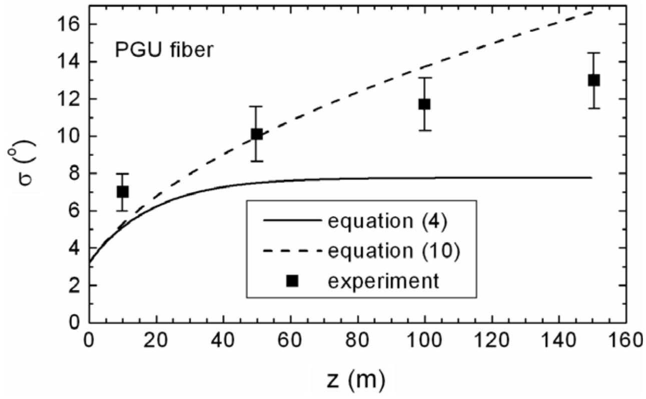

Figure 2. Standard deviation of the output angular power distribution as a function of fiber length in PGU fiber calculated by Gloge's function (4) (solid line) and by our recently proposed function (10) (dashed line). Filled squares represent experimental data.

The variance $\sigma^2(z)$ of the output angular power distribution (9) for the fiber length z can be calculated as [15]

$$\sigma^2(z) = \sigma_0^2 + 2Dz \tag{10}$$

which is of the same form as equation (7). In our previous work [11, 16, 17] we have shown that equation (10) can be employed for step-index plastic and glass optical fibers, both at short and long fiber lengths. Because equation (10) is a newer alternative to equation (4), the two are compared in the next paragraph.

One should mention here that the coefficient D is assumed to be constant in both methods [5, 11]. The coefficient D was assumed constant by many other authors as well [1, 5–7, 12, 13]. In fact, this assumption dominates in the literature and has been shown to give excellent agreement with experimental results (e.g. [8, 10, 11, 13, 14, 16–19]). On the other hand, Mateo *et al* [12] did model the mode coupling process with D a function of θ and they obtained the same widths of the output angular power distribution as with the θ-independent D.

3. Comparison of methods

To facilitate the comparison of results by equations (4) [5] and (10) [11], we have applied them both on the same Eska™ Premier GH4001 (GH) step-index optical fiber (by Mitsubishi) and PGU-FB1000 (PGU) step-index optical fiber (from Toray), which are the fibers used in an earlier experiment [12]. These fibers have core diameter d = 1 mm, numerical aperture NA = 0.5 (corresponding to inner critical angle of $\theta_c = 19.5°$) and the nominal attenuation is 0.15 dB m^{-1}. In modeling mode coupling by (4), Mateo *et al* [12] used the value of $D = 1.171 \times 10^{-4}$ rad^2 m^{-1} and $D = 2.271 \times 10^{-4}$ rad^2 m^{-1} for the coupling coefficient of the GH and PGU fiber, respectively. We adopt these values for this quantity that was assumed constant by many authors [1, 5–7, 12, 13]. The value of $A = 0.4025$ rad^{-2} m^{-1} for the GH fiber [18] and $A = 0.8054$ rad^{-2} m^{-1} for the PGU fiber we also adopt in this work.

Calculated by equations (4) and (10) separately, figures 1 and 2 show standard deviation of the output angular power distribution as a function of length of the GH and PGU fibers, respectively, for the centrally launched Gaussian beam with a standard deviation of $\sigma_0 = 3.2°$. (Mateo *et al* [12] used a laser diode with FWHM = 7.5° (σ_0 = 3.2°) in the parallel plane and FWHM = 30° (σ_0 = 12.7°) in the perpendicular plane.) A significant difference between the two curves in figures 1 and 2 is apparent, heightening the need for comparison of the corresponding functions (4) and (10).

Mateo *et al* [12] experimentally obtained that SSD in a GH fiber has been reached at z_s = 175 m. By solving the power flow equation (1), we have obtained that SSD is reached at z_s = 190 m while EMD is reached at L_c = 80 m. Calculated using equation (4), the standard deviation of the output angular power distribution for GH fiber at lengths of z = 10, 50 and 150 m is σ = 4.6°, 7.1° and 8.2°, respectively. The corresponding values calculated using equation (10), differ: σ = 4.6°, 8.1° and 13.4°. Respective values measured are $\sigma \approx$ 5°, 11° and 12° [12]. Calculated using equation (4), the standard deviation of the output angular power distribution for PGU fiber at lengths of z = 10, 50, 100 and 150 m are σ = 5.1°, 7.5°, 7.7° and 7.8°, respectively. The corresponding values calculated using equation (10) differ: σ = 5.3°, 10°, 13.7° and 16.6°. Respective values measured are $\sigma \approx$ 7°, 10.1°, 11.5° and 12.9°. It appears that for short fiber lengths (z = 10 m) when the beam width is small compared to the steady state width, equations (4) and (10) give the same result which is close to measured data.

On increasing fiber length, predictions by equation (10) match the experimental measurements better than those of equation (4). This is explained by the fact that (4) has been derived from the analytical solution (3) that ignored the boundary condition $P(\theta_c, z) = 0$ at $\theta = \theta_c$. In contrast, (10) has been derived from the solution (9) of equation (8) that needs not account for this boundary condition [15]. The influence of the boundary condition $P(\theta_c, z) = 0$ omitted from (10) increases with fiber length because the output angular power distribution broadens towards $\theta = \theta_c$ due to mode coupling. At long fiber lengths, the width is influenced

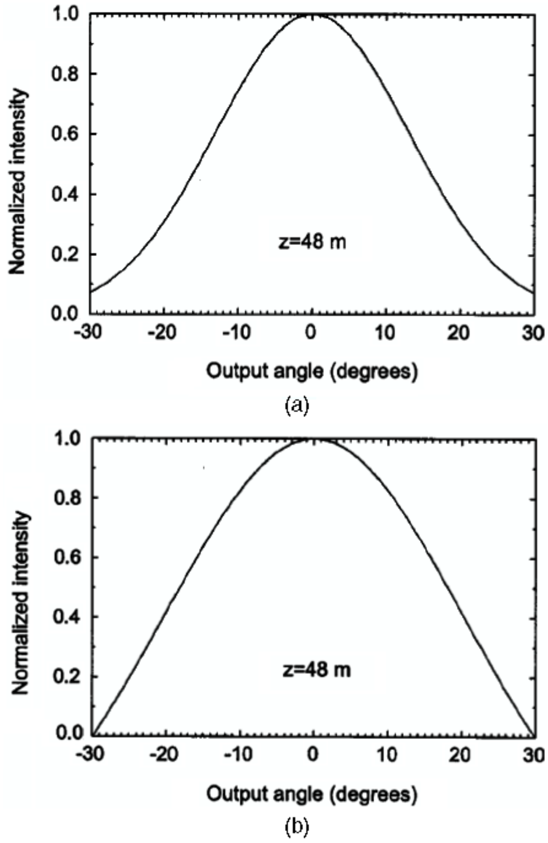

Figure 3. (a) Analytically obtained normalized steady state output angular power distribution as a solution of equation (1) at the end of the $z = 48$ m long GH fiber, obtained (a) ignoring the boundary condition $P(\theta_c, z) = 0$ and (b) including this boundary condition [19].

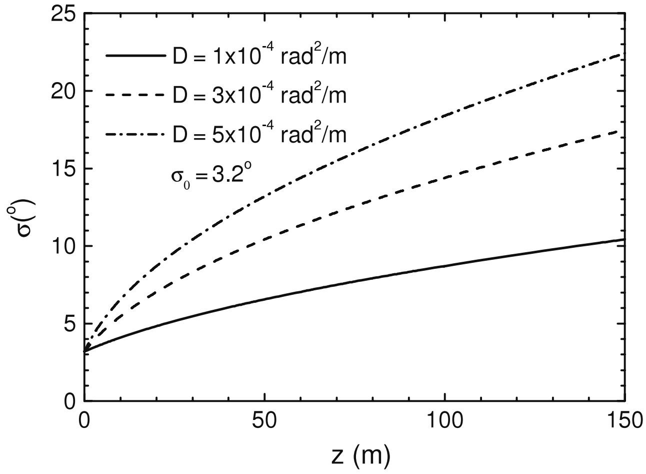

Figure 4. Evolution of the width of the output angular power distribution with fiber length z for various values of the diffusion coefficient D as a parameter.

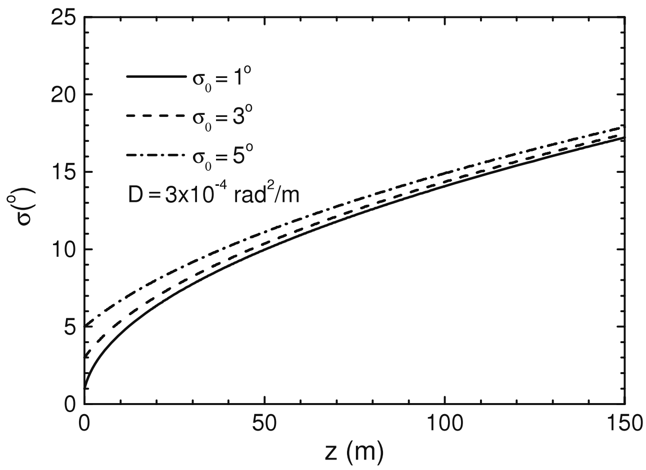

Figure 5. Evolution of the width of the output angular power distribution with fiber length z for various values of the width σ_0 of the launched beam as parameter.

apparently significantly. To illustrate this, the output angular power distribution is determined for the end of the $z = 48$ m long GH fiber, which is the fiber investigated earlier [14, 19]. Two analytical solutions of (1) (the normalized steady state output angular power distribution) are shown in figures 3(a) and (b). The one in figure 3(a) ignores the boundary condition $P(\theta_c, z) = 0$, and the one in figure 3(b) includes it. It can be observed that by taking into account the boundary condition $P(\theta_c, z) = 0$, a wider distribution is obtained ($\sigma = 16°$) compared to the case when this boundary condition is ignored ($\sigma = 13°$).

For parametric coefficients D and σ_0 in figures 4 and 5 (respectively), the influence of these parameters is shown in the variation of the width of the output angular power distribution with fiber length. A significant influence of the coupling coefficient D is apparent in figure 4, whereas that of the launch condition σ_0 diminishes with distance from the input fiber end (figure 5) and does not modify the shape of the curves for $\sigma(z)$.

4. Conclusion

We have tested two previously proposed functions for calculating the width of the output angular power distribution in two SI POFs. We found that the more recently proposed one matches better the experimental measurements. The other function underestimates the output width at longer fiber lengths, which is attributed to it not accounting for an appropriate boundary condition at $\theta = \theta_c$ (which is close to the critical angle of propagation). However, this missing boundary condition is irrelevant for short fibers because the launch is not normally at the critical angle $\theta = \theta_c$ and because not much power could be coupled to this angle by the coupling process that (in short fibers) is still at its very beginning. Hence, the two functions are equally accurate for short fibers.

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